# Continuous Multiple Importance Sampling

SIGGRAPH 2020

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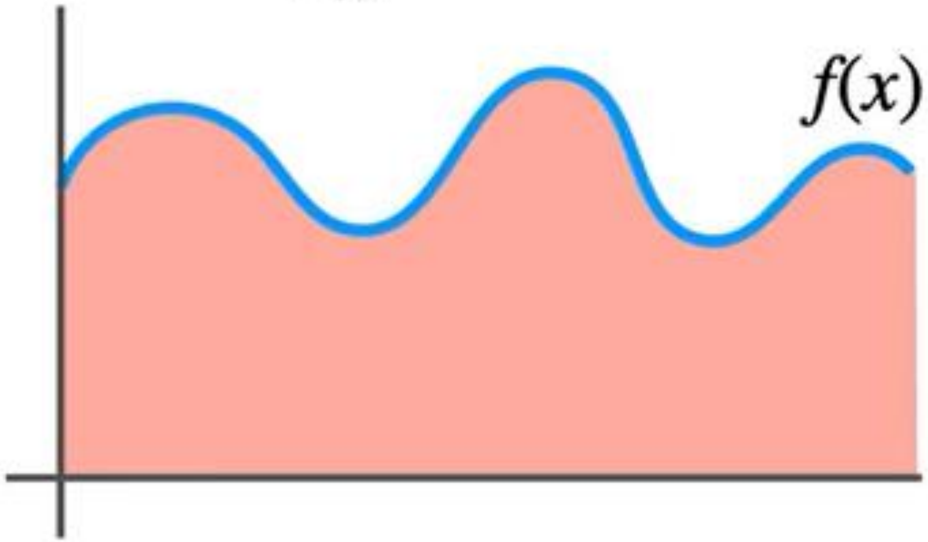
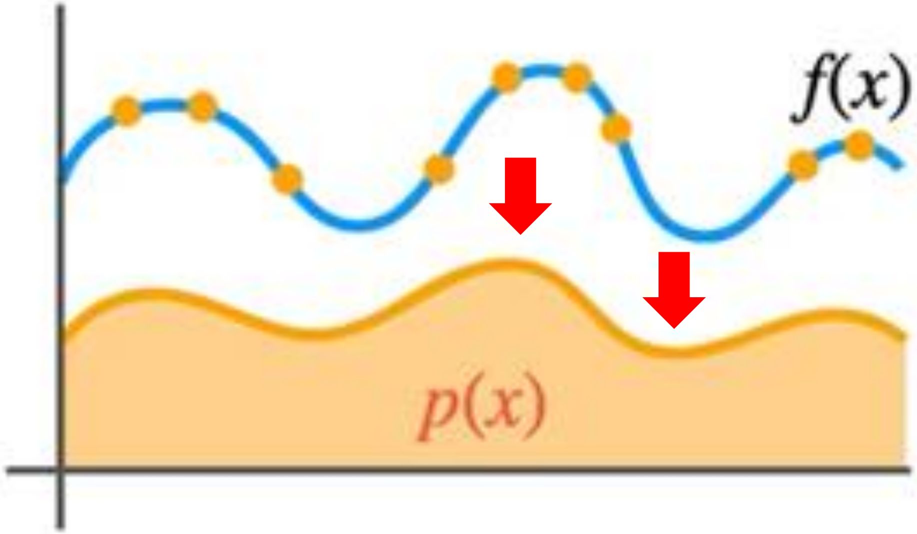
# Recap - Multiple Importance Sampling

$$\int_{\mathcal{X}} f(x)dx = I$$




Integration problem

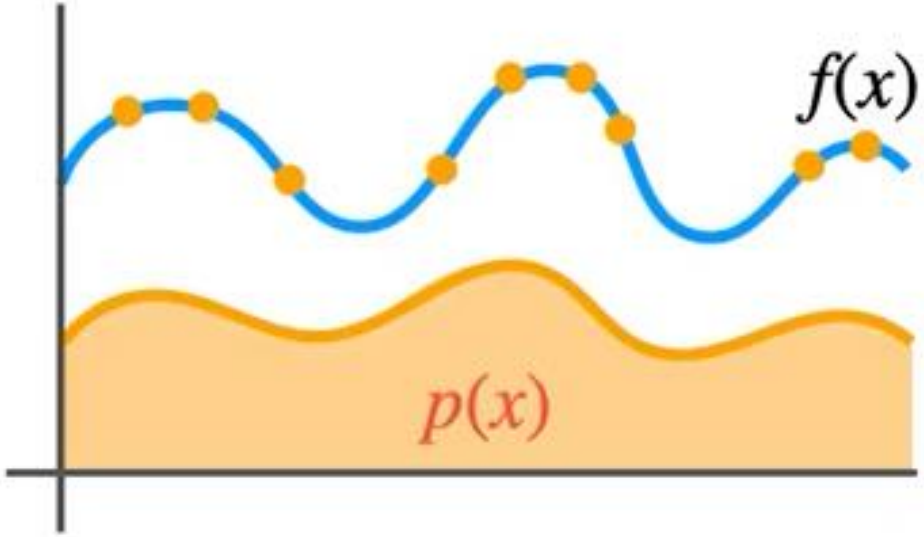
$$\frac{1}{n}\sum_{i=1}^{n}\frac{f(x_i)}{p(x_i)} = \langle I \rangle_n$$



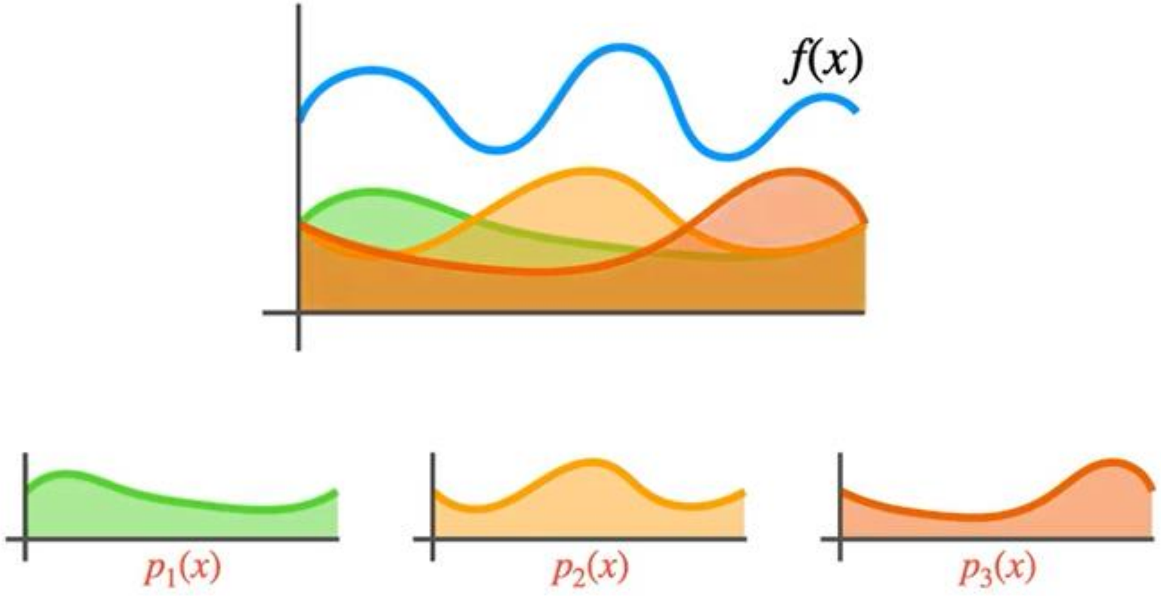

Importance Sampling

# Recap - Multiple Importance Sampling

$$\frac{1}{n}\sum_{i=1}^{n}\frac{f(x_i)}{p(x_i)} = \langle I \rangle_n$$




Importance Sampling



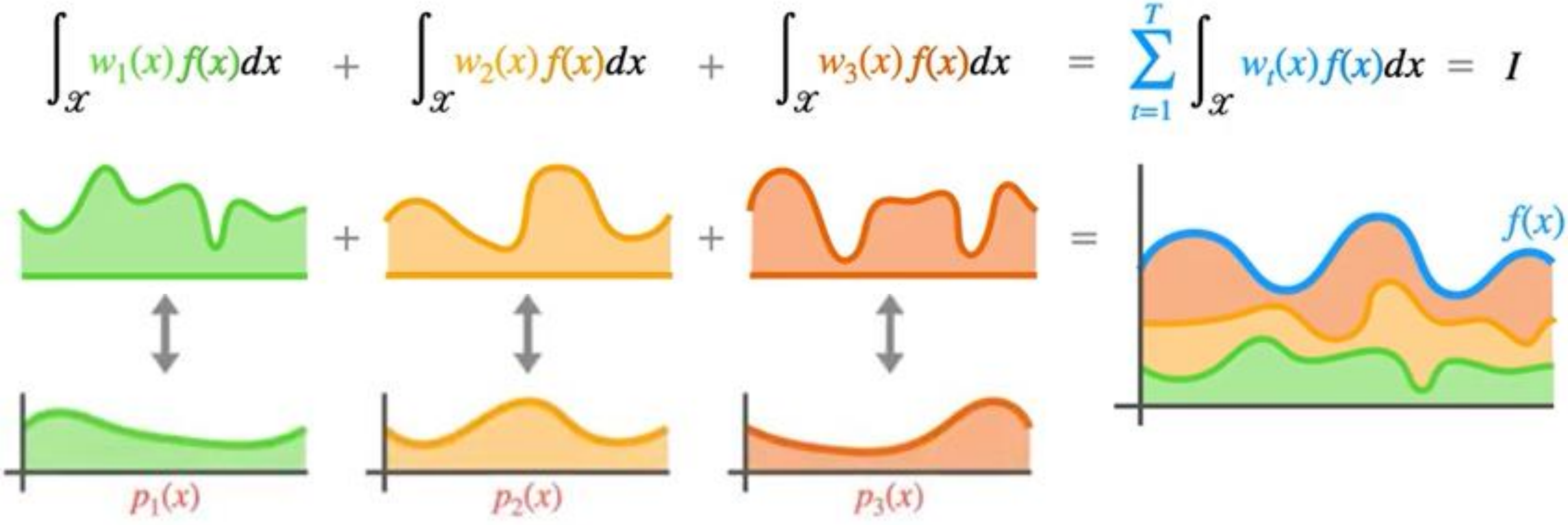

MIS

# Recap - Multiple Importance Sampling

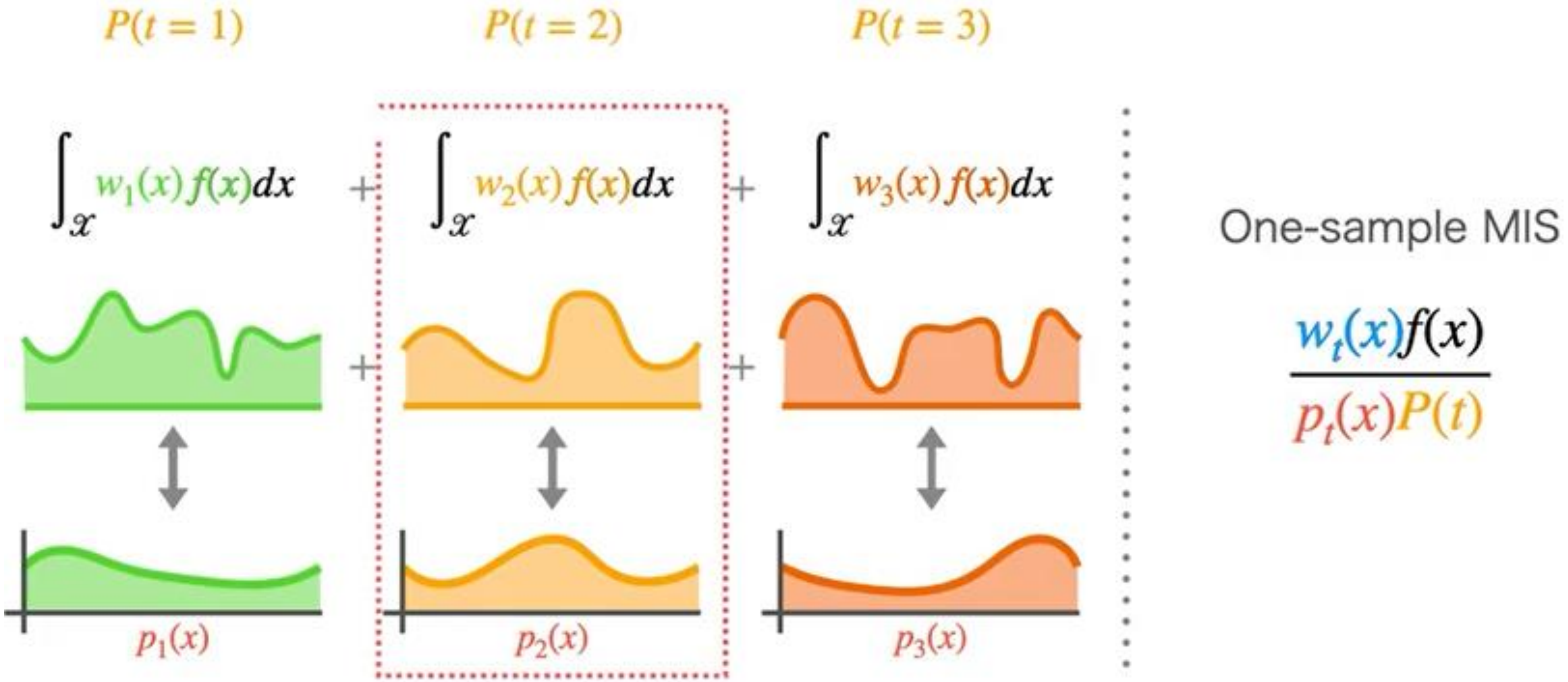
$$I = \int_X f(x)dx$$

$$\Rightarrow \int_X \underbrace{\sum_{t=1}^{T} w_t(x)}_{=1} f(x)\,\mathrm{d}x \quad \Rightarrow \quad \sum_{t=1}^{T} \int_X w_t(x) f(x)\,\mathrm{d}x$$

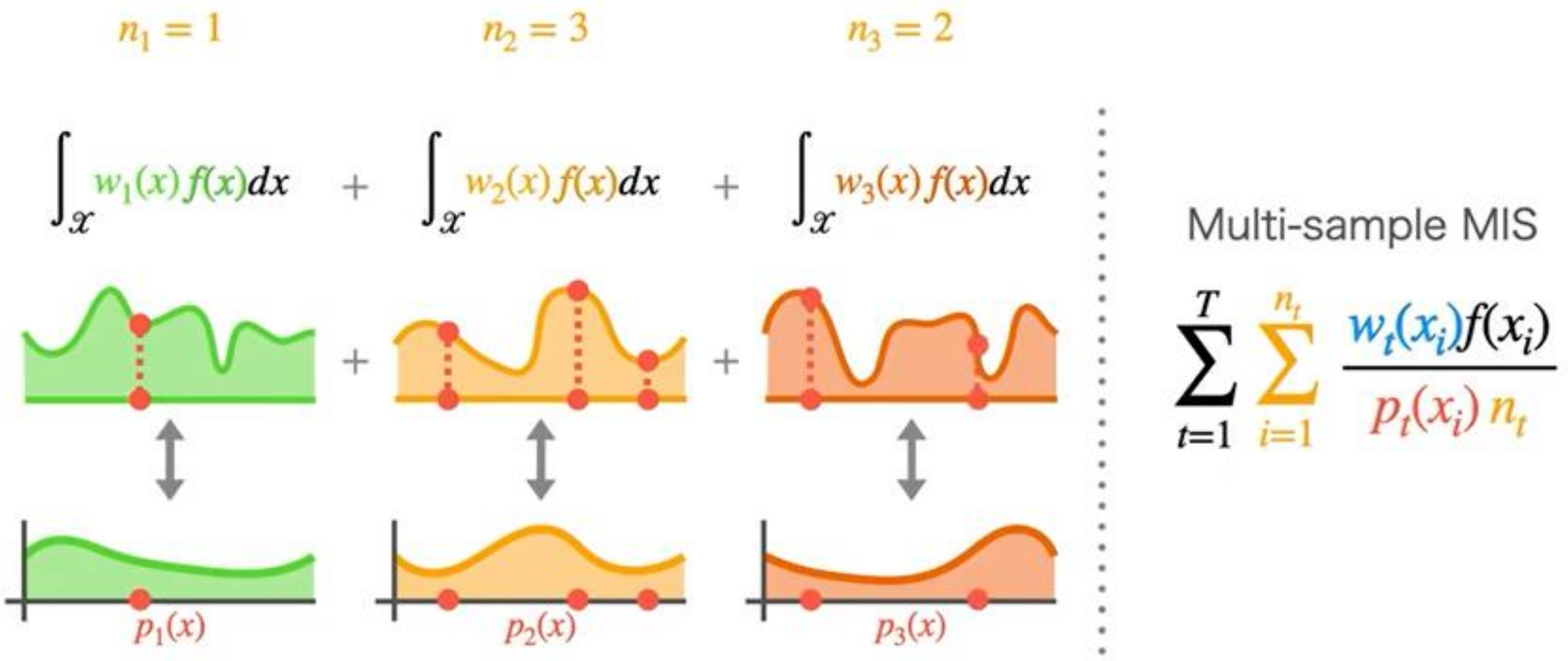
# Recap - Multiple Importance Sampling



# Recap - Multiple Importance Sampling



# Recap - Multiple Importance Sampling



# Recap – Balance Heuristic

Optimal MIS weights

THEOREM 5.2. *Let the column vector* $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)^\intercal$ *satisfy the system of linear equations*

$$\mathbf{A}\boldsymbol{\alpha} = \mathbf{b}, \qquad (12)$$

*where* **A** *and* **b** *are the technique matrix and the contribution vector, respectively. Then the weighting functions*

$$w_i^o(x) = \alpha_i \frac{p_i(x)}{f(x)} + \frac{n_i p_i(x)}{\sum_{j=1}^N n_j p_j(x)} \left(1 - \frac{\sum_{j=1}^N \alpha_j p_j(x)}{f(x)}\right) \qquad (13)$$

*minimize the functional* $V[w_1, \ldots, w_N]$.

$$\langle F \rangle^o = \sum_{i=1}^N \alpha_i + \frac{1}{M} \sum_{i=1}^N \sum_{j=1}^{n_i} \left(\frac{f(X_{ij})}{p_c(X_{ij})} - \frac{\sum_{k=1}^N \alpha_k p_k(X_{ij})}{p_c(X_{ij})}\right). \qquad (16)$$

One-sample MIS

$$\frac{w_t(x) f(x)}{p_t(x) P(t)}$$

↓

$$w_t(x) = \frac{p_t(x) P(t)}{\sum_{j=1}^T p_j(x) P(j)}$$

Multi-sample MIS

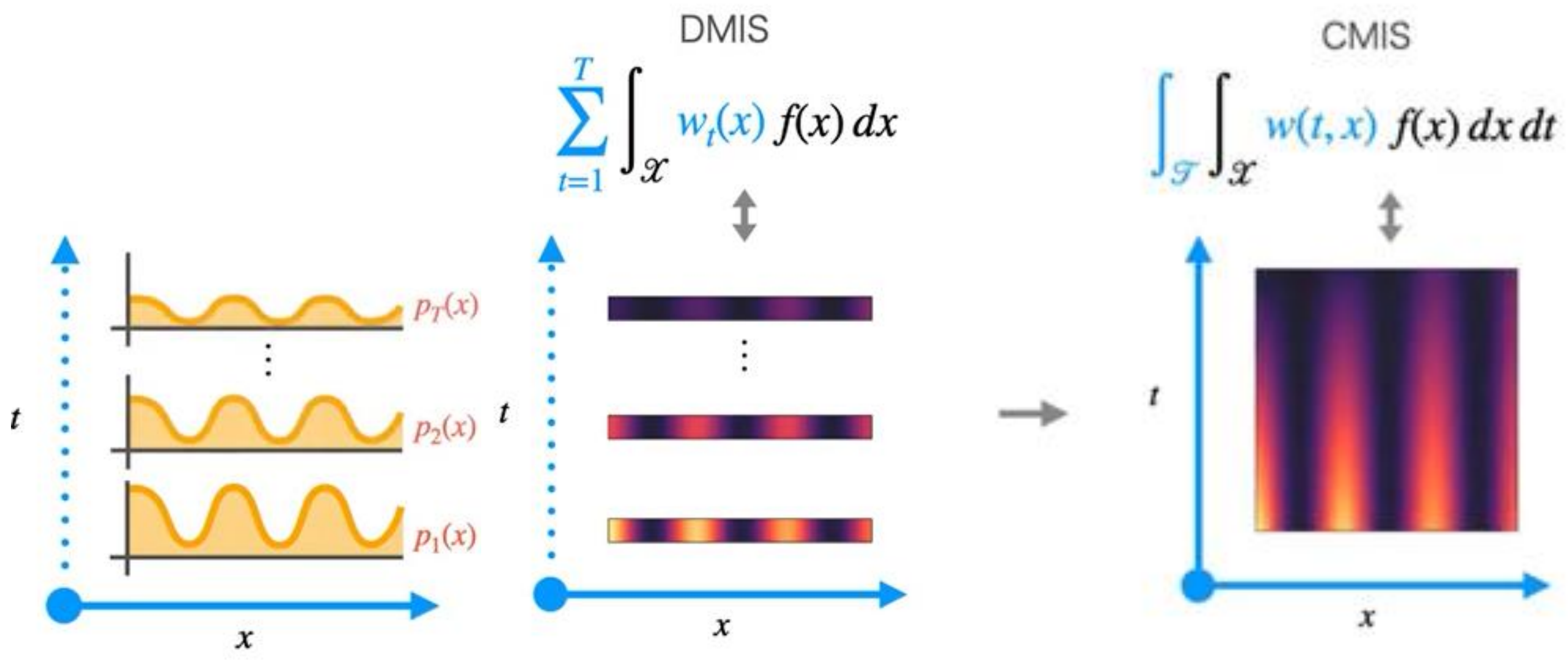
$$\sum_{t=1}^{T} \sum_{i=1}^{n_t} \frac{w_t(x_i) f(x_i)}{p_t(x_i)\, n_t}$$

↓

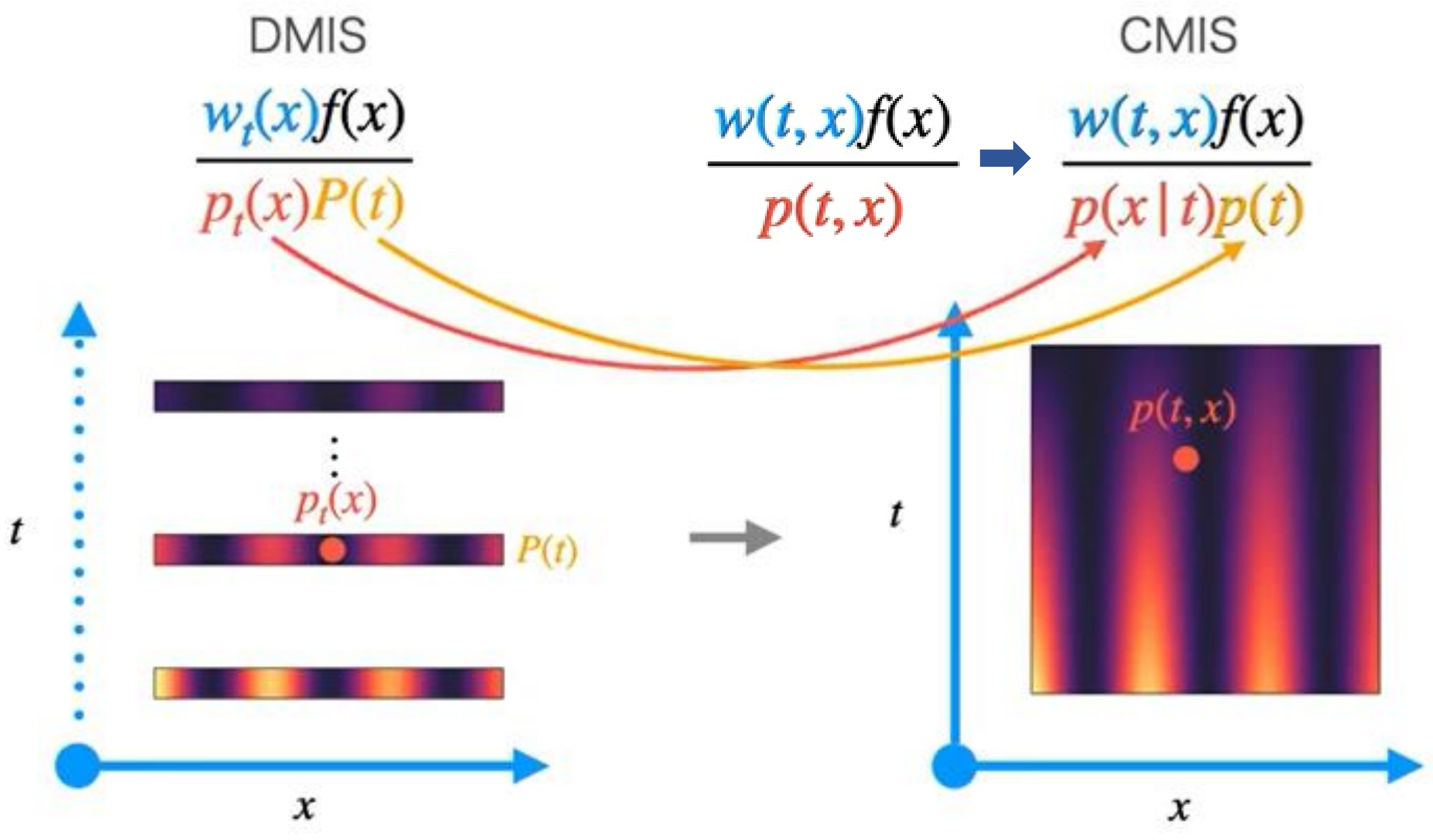
$$w_t(x) = \frac{p_t(x)\, n_t}{\sum_{j=1}^T p_j(x)\, n_j}$$

# Continuous MIS

# Continuous MIS



# Continuous MIS – one sampling



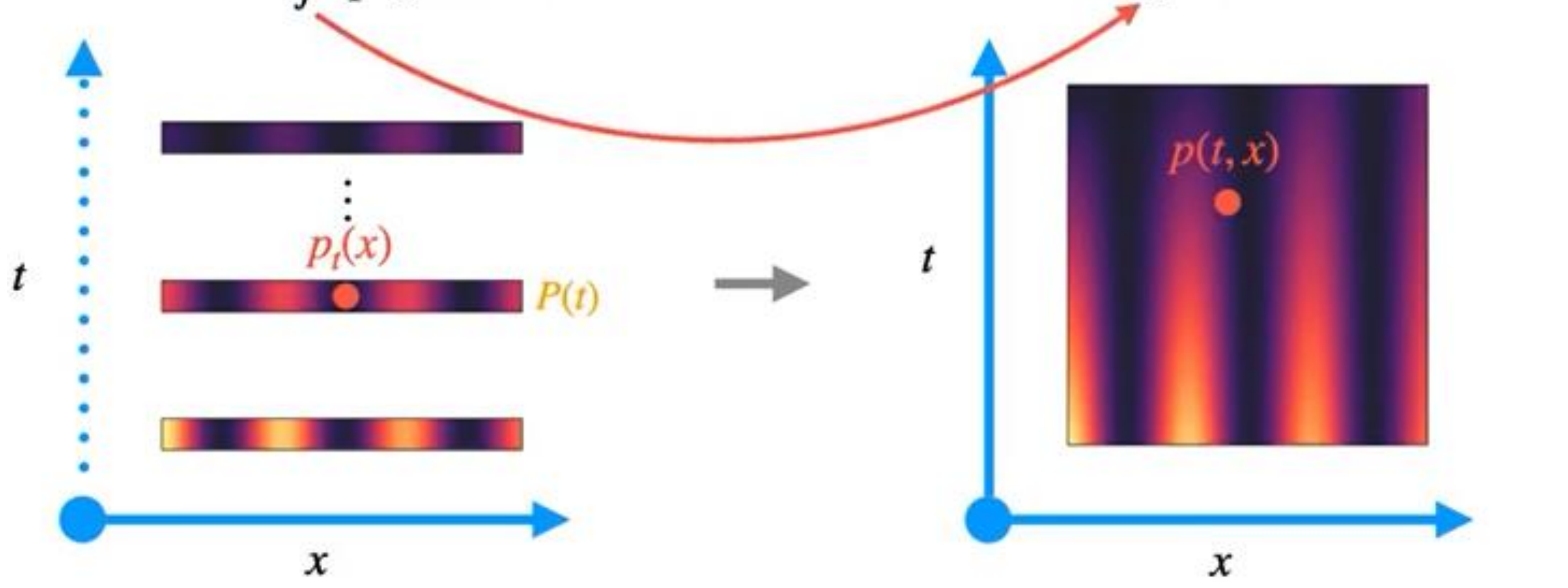
# Continuous MIS – balance heuristic

DMIS

$$w_t(x) = \frac{p_t(x)P(t)}{\sum_{j=1}^{T} p_j(x)P(j)}$$

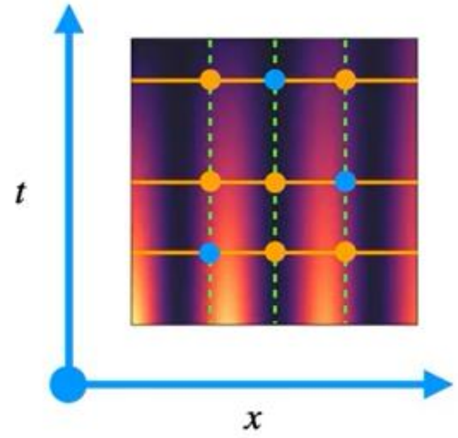
CMIS

$$w(t,x) = \frac{p(x\,|\,t)p(t)}{\int_{\mathcal{T}} p(x\,|\,t')p(t')\,dt'}$$



# Stochastic MIS

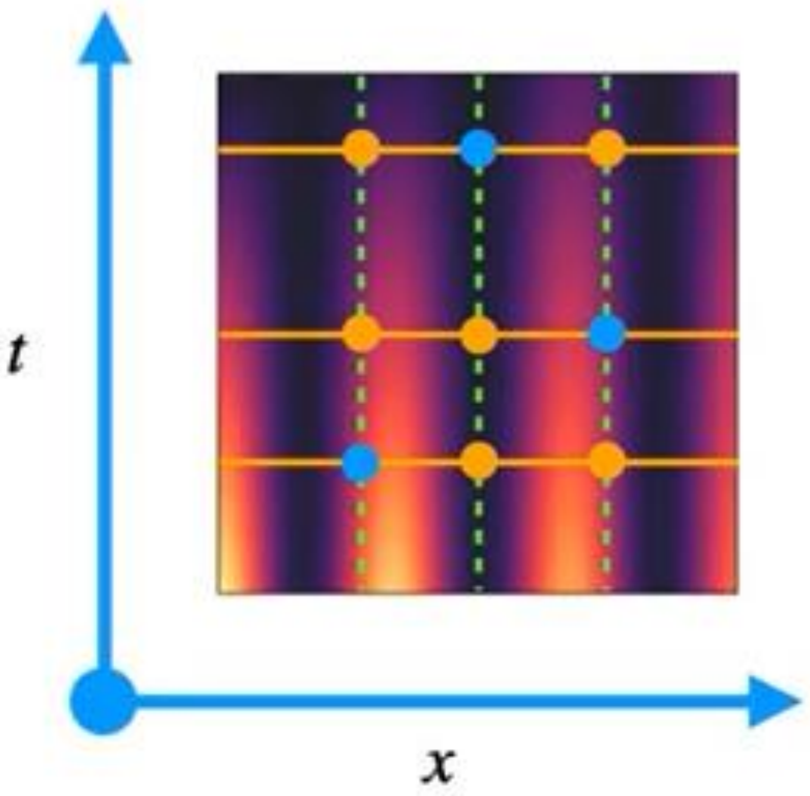
$$w(t,x) = \frac{p(x \mid t)p(t)}{\int_{\mathcal{T}} p(x \mid t')p(t')\,dt'}$$




$$w(t,x) = \frac{p(x \mid t)p(t)}{\frac{1}{T}\sum_{j=1}^{T} \frac{p(x \mid t_j)p(t_j)}{p(t_j)}}$$

$$w(t,x) = \frac{p(x \mid t)p(t)}{\frac{1}{T}\sum_{j=1}^{T} p(x \mid t_j)}$$

# Stochastic MIS




**Stochastic MIS**

One sample per technique

$$\sum_{i=1}^{T} \frac{p(x_i \mid t_i)}{\sum_{j=1}^{T} p(x_i \mid t_j)} \frac{f(x_i)}{p(x_i \mid t_i)}$$

$\neq$

Multi-sample MIS

$$\sum_{t=1}^{T} \sum_{i=1}^{n_t} \frac{w_t(x_i) f(x_i)}{p_t(x_i)\, n_t}$$

↓

$$w_t(x) = \frac{p_t(x)\, n_t}{\sum_{j=1}^{T} p_j(x)\, n_j}$$

# Application #1

Path Reuse

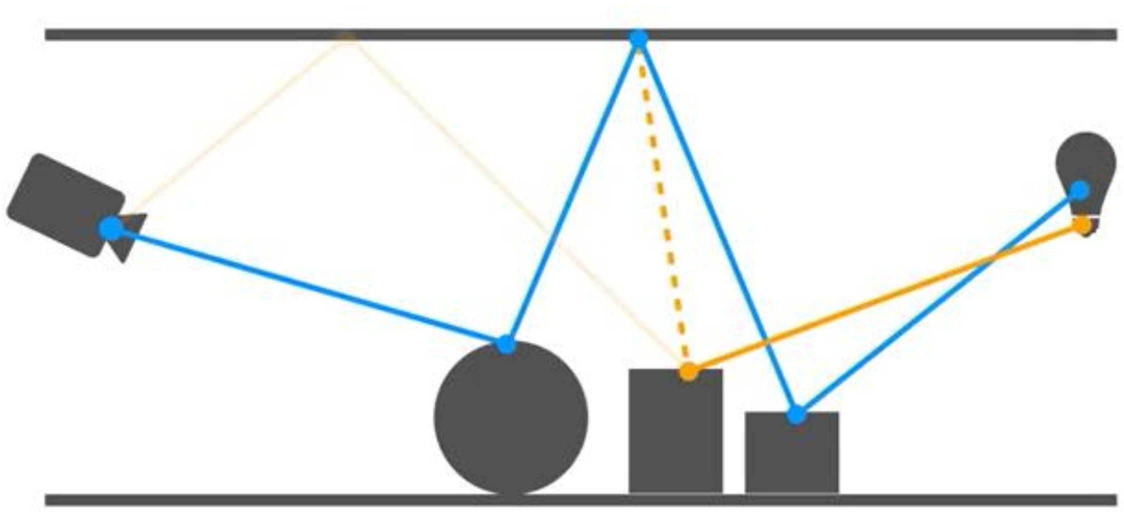
# Previous work – path filtering

*Alexander Keller, Ken Dahm, and Nikolaus Binder. 2014. Path Space Filtering (SIGGRAPH '14).*

## Motive

1. To reduce noise of random sampling in path tracing, we want to sample more paths, but this can be quite expensive.

2. To reduce these costs, we use other existing paths by reconnecting



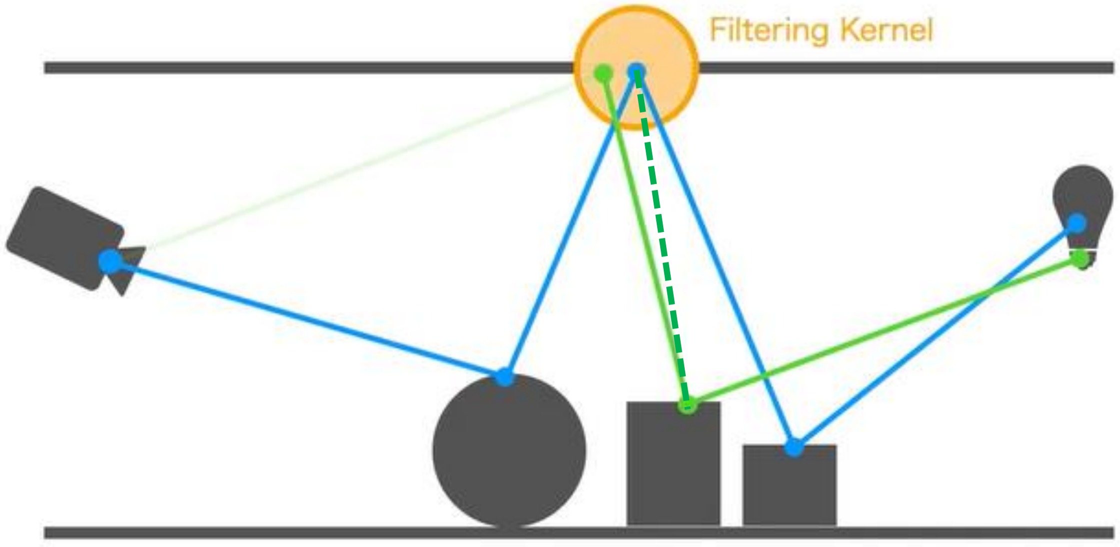
# Previous work – path filtering

*Alexander Keller, Ken Dahm, and Nikolaus Binder. 2014. Path Space Filtering (SIGGRAPH '14).*

## Algorithm

1. Generate some filtering kernels while path sampling

2. When any generated path reaches the filtering kernel, the remaining path is set to the existing path that generated that filtering kernel.



# Previous work – path filtering

*Alexander Keller, Ken Dahm, and Nikolaus Binder. 2014. Path Space Filtering (SIGGRAPH '14).*

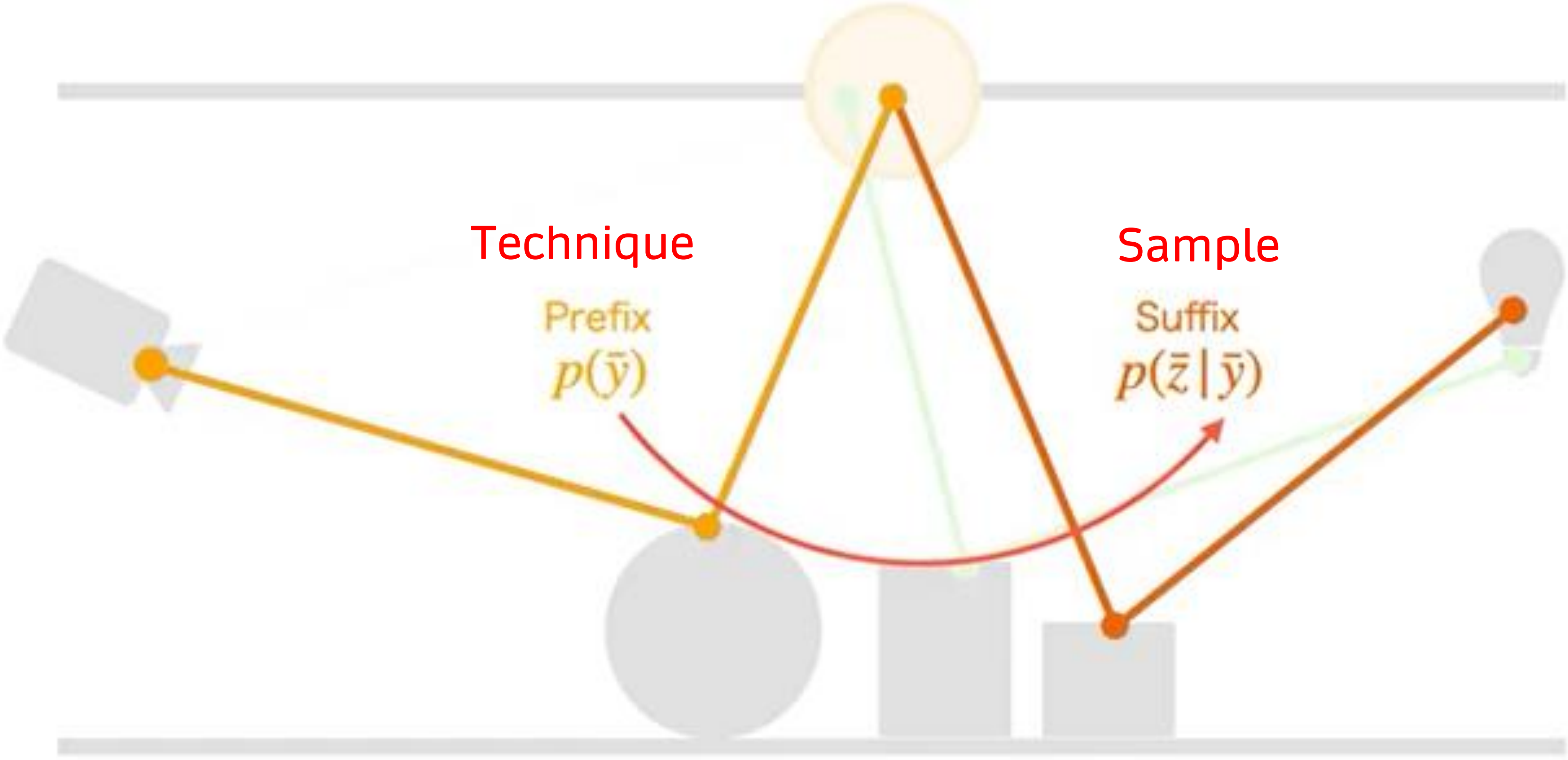
Problem

1. Bias

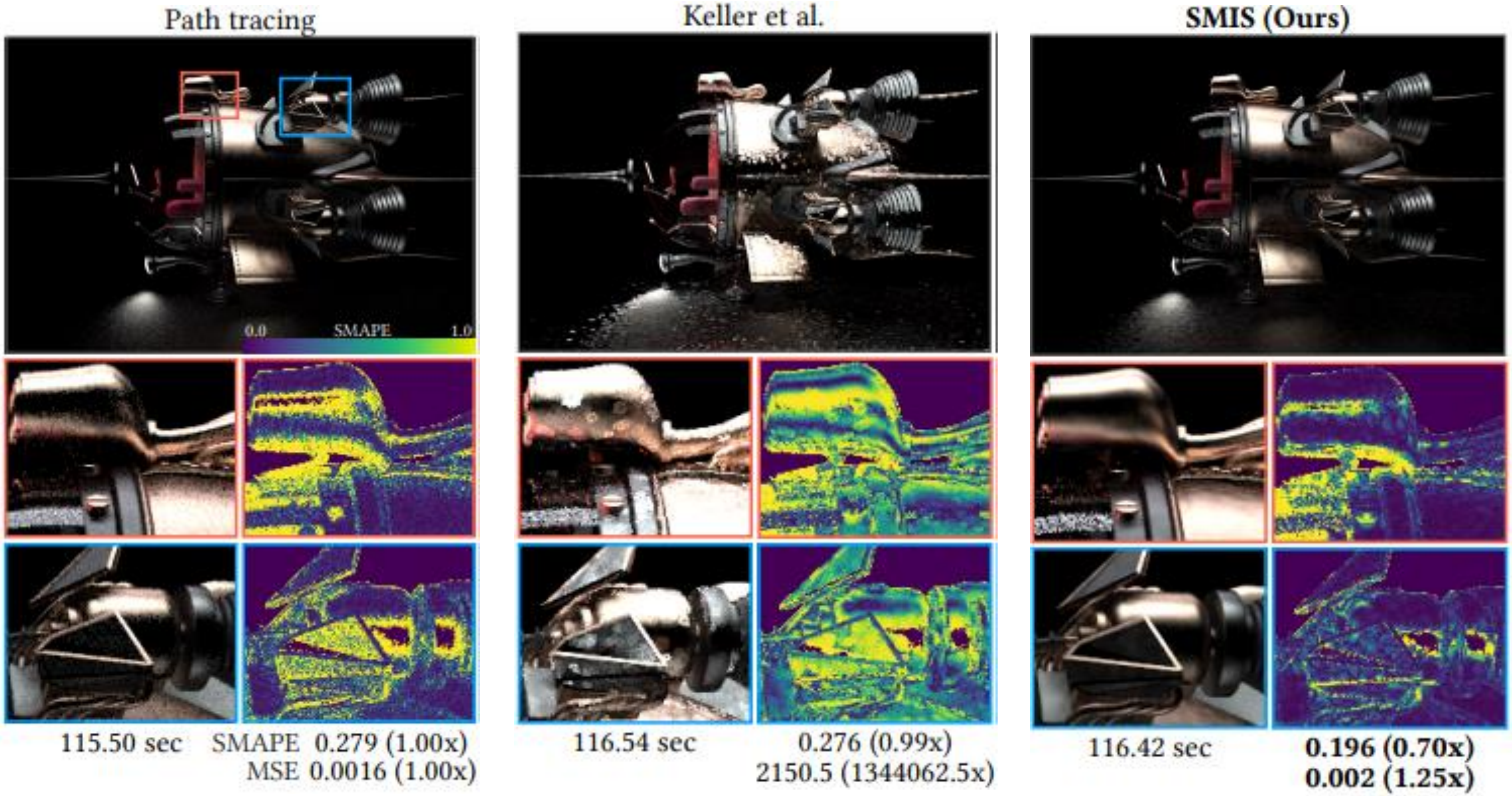
2. Works well on diffuse materials but it has some

difficulty with glossy surfaces in detail geometry

# Path filtering - CMIS



# Path filtering - CMIS



# Application #2

## Spectral Rendering

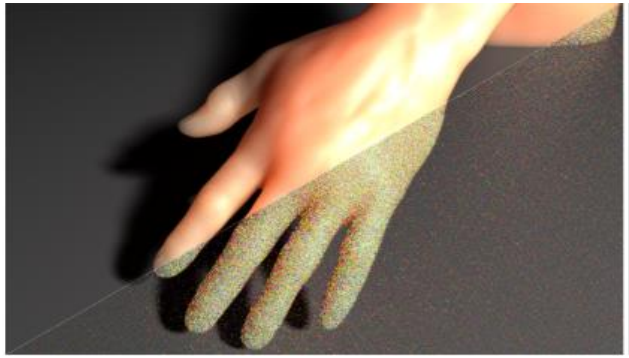
# Previous work – spectral rendering

*Alexander Wilkie, Sehera Nawaz, Marc Droske, Andrea Weidlich, and Johannes Hanika. 2014. Hero Wavelength Spectral Sampling.*

## Motive

1. Overcome the limitations of tri-stimulus (RGB) rendering

2. Extend the path integral over the visible rays wavelength domain



Single wavelength sampling

Spectral rendering

# Previous work – spectral rendering

*Alexander Wilkie, Sehera Nawaz, Marc Droske, Andrea Weidlich, and Johannes Hanika. 2014. Hero Wavelength Spectral Sampling.*

## Algorithm (Hero MIS)

1. Sample a wavelength

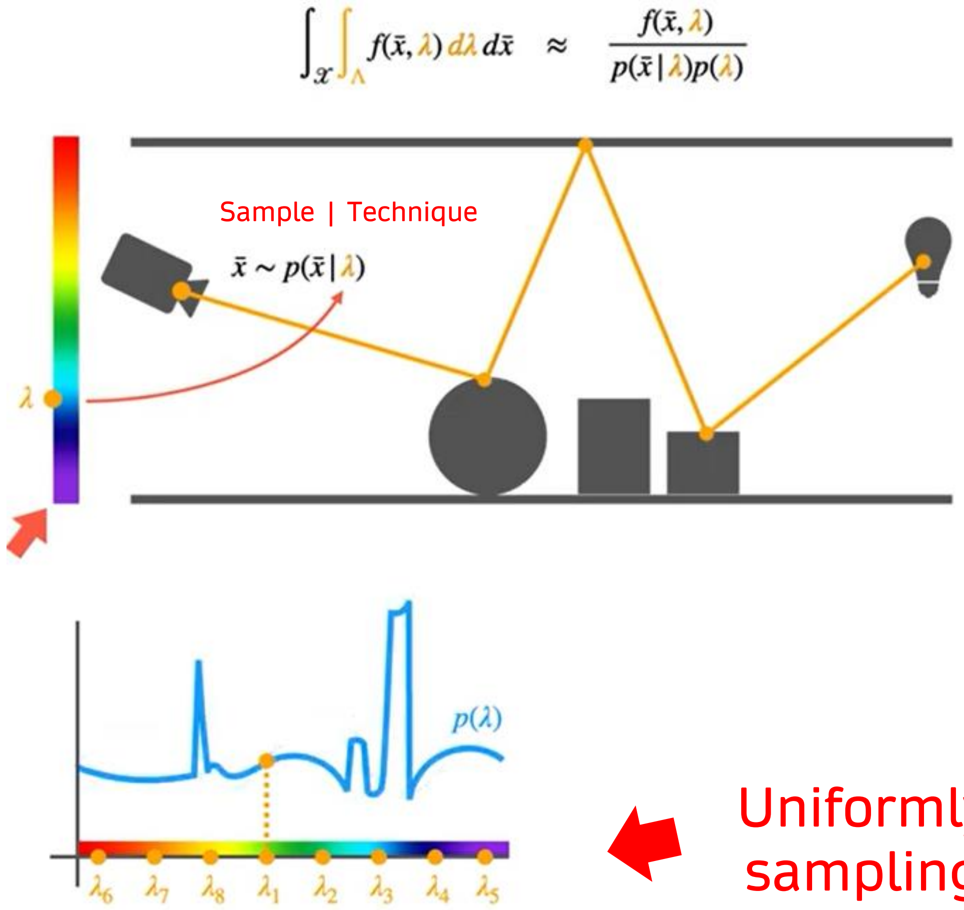
2. Given a sampled wavelength, sample n

wavelengths which are uniformly spaced

3. N spectral paths are sampled

{x, $\lambda$1}, {x, $\lambda$2}, .. , {x, $\lambda$n}.

4. This corresponds to having n sampling DMIS




Uniformly sampling

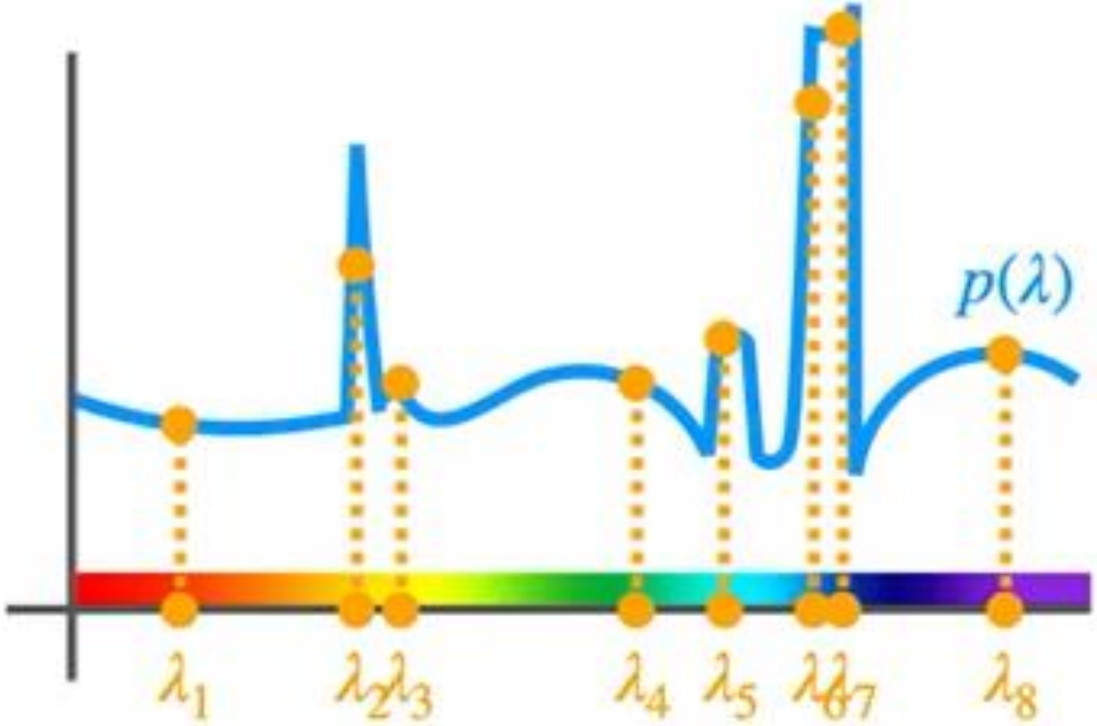
# Previous work – spectral rendering

*Alexander Wilkie, Sehera Nawaz, Marc Droske, Andrea Weidlich, and Johannes Hanika. 2014. Hero Wavelength Spectral Sampling.*

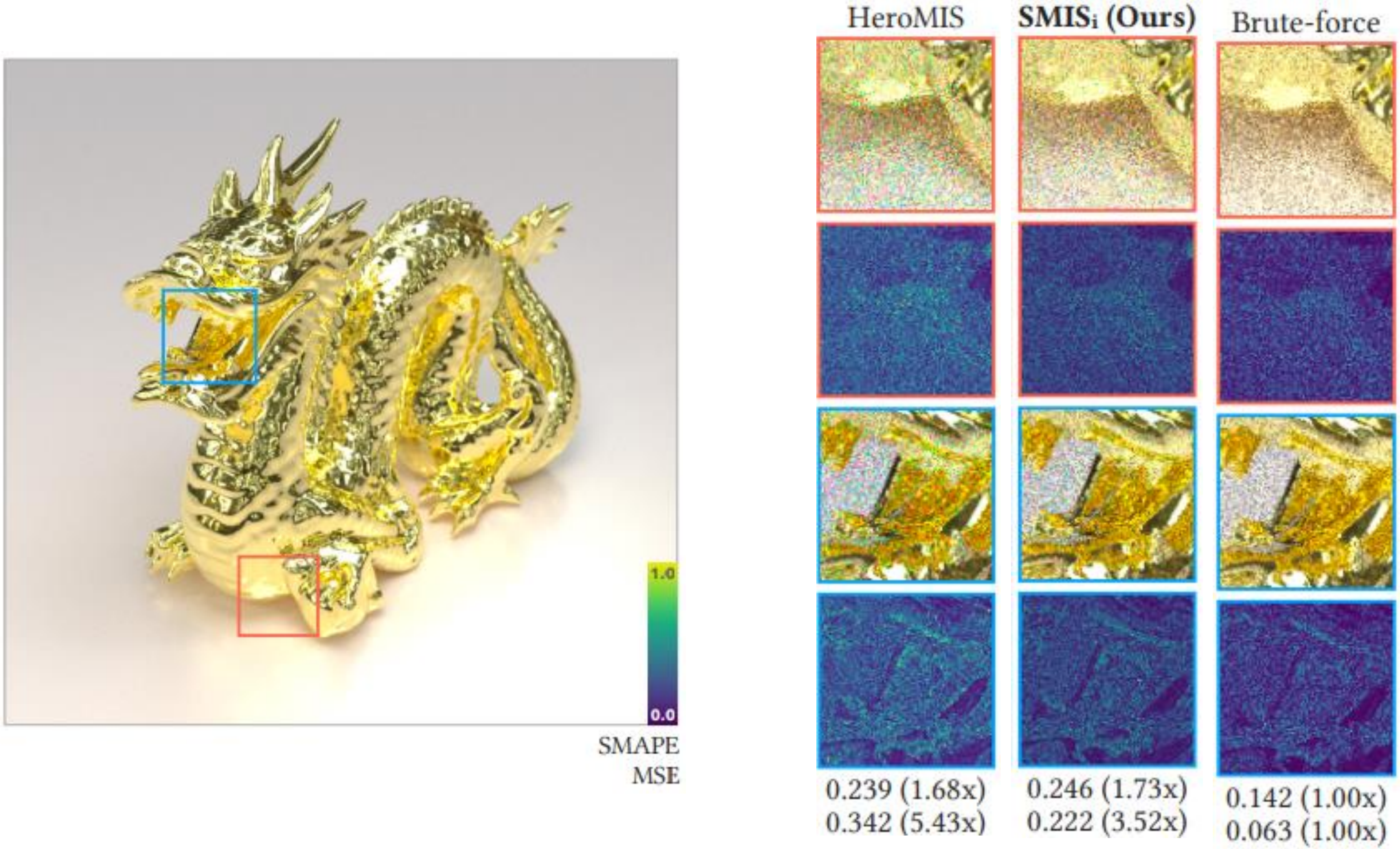
## Problem

- Uniform wavelength spacing is sub-optimal for spectral power distributions that concentrate energy at a few peaks (e.g., fluorescent lights).

# Spectral rendering– CMIS



# Spectral rendering– CMIS



# Summary

- Recap - Multiple Importance Sampling (MIS)
- Continuous Multiple Importance Sampling (CMIS)
- Stochastic Multiple Importance Sampling (SMIS)
- Application
    - Path reuse
    - Spectral rendering

# Thank you